Fifth Grade Unit 6 Mathematics

Dear Parents,

The Mathematics Georgia Standards of Excellence (MGSE), present a balanced approach to mathematics that stresses understanding, fluency, and real world application equally. Know that your child is not learning math the way many of us did in school, so hopefully being more informed about this curriculum will assist you when you help your child at home.

Below you will find the standards from Unit Six in bold print and underlined. Following each standard is an explanation with student examples. Please contact your child's teacher if you have any questions.

OA.2 Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them

This standard refers to expressions. Expressions are not equations. Expressions are a series of numbers and symbols (+, -, x, ÷) without an equal sign. Equations, however, have an equal sign.

Example:

- 4(5 + 3) is an expression.
- When we compute 4(5 + 3), we are evaluating the expression. The expression's value is 32.
- 4(5 + 3) = 32 is an equation.

This standard calls for students to verbally describe the relationship between expressions without actually calculating them. This standard does not include the use of variables, only numbers and symbols for operations.

Example:

- Write an expression for "double five and then add 26."

Student: *(2 × 5) + 26*

- Describe how the expression 5(10 × 10) relates to 10 × 10.

Student: The value of the expression 5(10 × 10) is 5 times larger than the expression 10 × 10. I know that because 5(10 × 10) means that I have 5 groups of (10 × 10).

NBT.1 Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.

This standard calls for students to reason about the magnitude of numbers. Students should work with the idea that the tens place is ten times as much as the ones place, and the ones place is $1/10^{th}$ the size of the tens place.

Example:

- The value of the 2 in the number 542 is different from the value of the 2 in 324. The 2 in 542 represents 2 ones or 2, while the 2 in 324 represents 2 tens or 20. Since the 2 in 324 is one place to the left of the 2 in 542, the value of the 2 is 10 times greater.
- Meanwhile, the 4 in 542 represents 4 tens or 40 and the 4 in 324 represents 4 ones or 4. Since the 4 in 324 is one place to the right of the 4 in 542, the value of the 4 in the number 324 is $1/10^{th}$ of the value in the number 542.

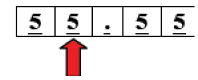
Example:

- A student thinks, "I know that in the number 5555, the 5 in the tens place (55<u>5</u>5) represents 50 and the 5 in the hundreds place (5<u>5</u>55) represents 500. So, a 5 in the hundreds place is ten times as much as a 5 in the tens place, or a 5 in the tens place is $1/10^{th}$ of the value of a 5 in the hundreds place.

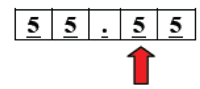
- To extend this understanding of place value to their work with decimals, students use a model of one unit; they cut it into 10 equal pieces, shade in, or describe $1/10^{th}$ of that model using fractional language. ("This is 1 out of 10 equal parts so it is 1/10. I can write this using 1/10 or 0.1.") They repeat the process by finding 1/10 of a 1/10 (e.g., dividing 1/10 into 10 equal parts to arrive at 1/100 or 0.01) and can explain their reasoning: "0.01 is 1/10 of 1/10 thus is 1/100 of the whole unit."

Additional Examples:

In the number 55.55, each digit is 5, but the value of the digits is different because of the placement. The 5 that the arrow points to is 1/10 of the 5 to the left and 10 times the 5 to the right. The 5 in the ones place is 1/10 of 50 and 10 times 5/10.



The 5 that the arrow points to is 1/10 of the 5 to the left and 10 times the 5 to the right. The 5 in the tenths place is 10 times five hundredths.



NBT.2 Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.

This standard includes multiplying or dividing by multiples of 10 and powers of 10, including 10^2 which is $10 \times 10 = 100$, and 10^3 which is $10 \times 10 \times 10 = 1{,}000$.

Examples: $25 \times 10^3 = 25 \times (10 \times 10 \times 10) = 25 \times 1{,}000 = 25{,}000$; $25 \div 10^3 = 25 \div (10 \text{ x } 10 \text{ x } 10) = 25 \div 1{,}000 = 0.025$

- Students should reason that the exponent indicates how many times 10 is multiplied by itself. Multiplying by that power of 10 then increases the place value of the digits in the original number while dividing by that power of 10 decreases the place value of the digits in the original number. For example:
  $3 \text{ x } 10^2$ is 3 x (10 x 10) or 3 x 100 which is 300; and $3 \div 10^2$ is $3 \div (10 \text{ x } 10)$ or $3 \div 100$ which is 0.03.

- Students might write:
  - $36 \times 10^1 = 36 \times 10 = 360$
  - $36 \times 10^2 = 36 \times (10 \times 10) = 3600$
  - $36 \div 10^1 = 36 \div 10 = 3.6$
  - $36 \div 10^2 = 36 \div (10 \text{ x } 10) = 0.36$

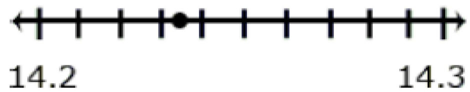
- Students might think and/or say:
  - I noticed that every time I multiplied by 10, I added a zero to the end of the number. That makes sense, because each digit's value became 10 times larger. To make a digit 10 times larger, I have to move it one place value to the left.
  - I noticed that every time I divided by 10, the number became smaller by 1/10. That makes sense, because each digit's value became 1/10 smaller. To make a digit 1/10 smaller, I have to move it one place value to the right.
  - Students should be able to use the same type of reasoning as above to explain why the following multiplication and division problems by powers of 10 make sense.
    $523 \times 10^3 = 523{,}000$ The place value of 523 is increased by 3 places.
    $5.223 \times 10^2 = 522.3$ The place value of 5.223 is increased by 2 places.
    $52.3 \div 10^1 = 5.23$ The place value of 52.3 is decreased by one place.

NBT.4 Use place value understanding to round decimals to any place.

This standard calls for students to use their understanding of place value and number sense to explain and reason about rounding. Students should "round by reason" rather than by a "rote rule". They should have numerous experiences using a number line to support their work with rounding.

Example: Round 14.235 to the nearest tenth.

- Students recognize that the possible answer must be in tenths and is between 14.2 or 14.3. They can then identify that 14.235 is closer to 14.2 (14.20) than to 14.3 (14.30).



- Students should use benchmark numbers to support this work. Benchmarks are numerical reference points for comparing and rounding numbers. (0, 0.5, 1, 1.5 are examples of benchmark numbers.)

NBT.5 Fluently multiply multi-digit whole numbers using the standard algorithm.

This standard builds upon students' work with multiplying numbers in 3rd and 4th grade. In 4th grade, students developed understanding of multiplication using various strategies. While learning the standard algorithm is the focus, alternate strategies are also appropriate to help students develop conceptual understanding. Students' work is limited to multiplying three-digit by two-digit numbers.

******Primary focus on 4 digit by 2 digit multiplication in this unit.******

Examples of alternate strategies:

- There are 225 dozen cookies in the bakery. How many cookies are there?

Student 1

225 × 12

I broke 12 up into 10 and 2.

225 × 10 = 2,250
225 × 2 = 450

2,250 + 450 = 2,700

Student 2

225 × 12

I broke 225 up into 200 and 25.

200 × 12 = 2,400

I broke 25 up into 5 × 5, so I had 5 × 5 × 12 or 5 × (5 x 12).

5 × 12 = 60 and 60 × 5 = 300

Then I added 2,400 and 300.

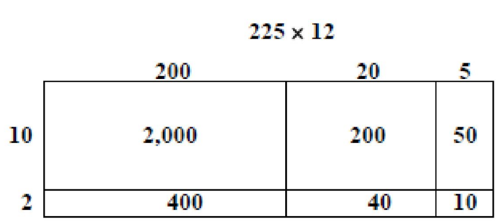
2,400 + 300 = 2,700

Student 3

I doubled 225 and cut 12 in half to get 450 × 6. Then I doubled 450 again and cut 6 in half to 900 × 3.

900 × 3 = 2,700

- Draw an array model for 225 × 12



2,000
400
200
40
50
+ 10
2,700

<u>NBT.6 Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.</u>

******Primary focus on 4 digit by 1 digit division in this unit.******

This standard references various strategies for division. Division problems can include remainders. This standard extends students' prior experiences with strategies, illustrations, and explanations. When the two-digit divisor is a "familiar" number, a student might decompose the dividend using place value.

Example:

- There are 1,716 students participating in Field Day. They are put into teams of 16 for the competition. How many teams will there be? If you have left over students, what do you do with them?

Student 1

1,716 ÷ 16

There are <u>100</u> 16's (1,600) in 1,716.

1,716 – 1,600 = 116

I know there are at least <u>6</u> 16's (96) in 116.

116 – 96 = 20

There is still <u>1</u> more 16 in 20.

20 – 16 = 4

There are 107 (100 + 6 + 1) teams with 16 students with 4 students left over. If we put the extra students on different teams, 4 teams will have 17 students.

Student 2

1,716 ÷ 16

There are 100 16's in 1,716.

Ten groups of 16 is 160. That's too big. Half of that is 80, whic is 5 groups of 16.

I know that 2 groups of 16 is 32

I have 4 students left over.

1,716 – 1,600	100
116 – 80	5
36 – 32	2
4	

There are 100 + 5 + 2 or 107 teams of 16 students with 4 students left over. Those students could be added to four of the teams.

Example: 9984 ÷ 64

- A partial quotient model for division is shown below. As the student uses the partial quotient model, he/she keeps track of how much of the 9984 is left to divide.

64)9984	
–6400	(100 × 64)
3584	
–3200	(50 × 64)
384	
–320	(5 × 64)
64	
–64	(1 × 64)
0	

There were 100 + 50 + 5 + 1 or 156 sets of 64 in 9,984. The final quotient for 9984 ÷ 64 is 156 with no remainder.

NBT.7 Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

******Primary focus on multiplication and division in this unit.******

In 5th grade, students begin adding, subtracting, multiplying, and dividing decimals. This work should focus on concrete models and pictorial representations, rather than relying solely on the algorithm. The use of symbolic notations involves having students record the answers to computations (2.25 × 3 = 6.75), but this work should not be done without models or pictures. This standard includes students' reasoning and explanations of how they use models, pictures, and strategies.

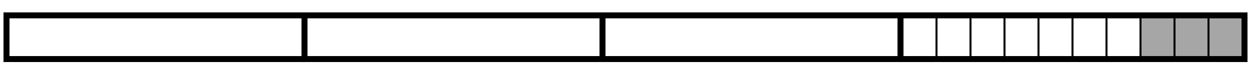
Before students are asked to give exact answers, they should estimate answers based on their understanding of operations and the value of the numbers.

Examples:

- **3.6 + 1.7**
  A student might estimate the sum to be larger than 5 because 3.6 is more than 3½ and 1.7 is more than 1½.
- **5.4 – 0.8**
  A student might estimate the answer to be a little more than 4.4 because a number less than 1 is being subtracted.
- **6 × 2.4**
  A student might estimate the answer to be between 12 and 18 since 6 × 2 is 12 and 6 × 3 is 18. Another student might give an estimate of a little less than 15 because s/he figures the answer to be very close, but smaller than 6 × 2½ and think of 2½ groups of 6 as 15 (2 groups of 6) + 3 (½ of a group of 6).

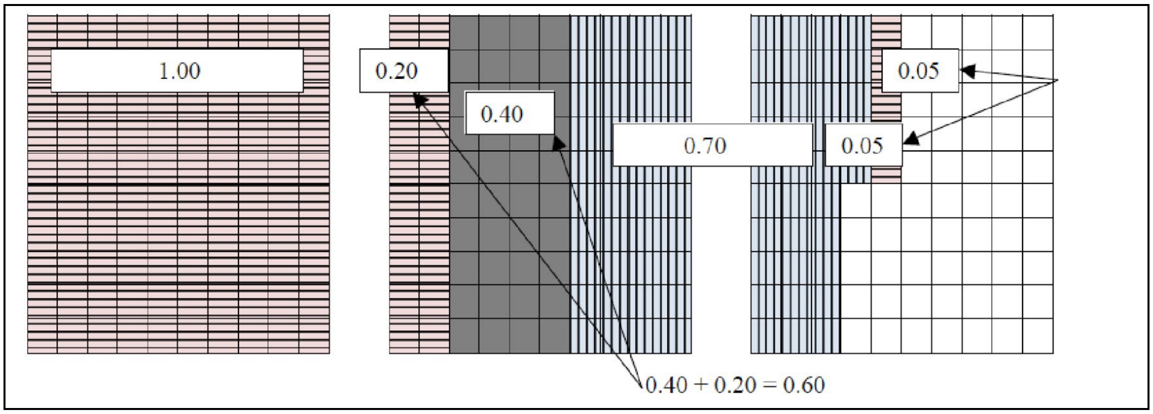
Students should be able to express that when they add decimals they add tenths to tenths and hundredths to hundredths. So, when they are adding in a vertical format (numbers beneath each other), it is important that they write numbers with the same place value beneath each other. This understanding can be reinforced by connecting addition of decimals to their understanding of addition of fractions. Adding fractions with denominators of 10 and 100 is a standard in fourth grade.

Example: **4 - 0.3 (**3 tenths subtracted from 4 wholes)
One of the wholes must be divided into tenths.

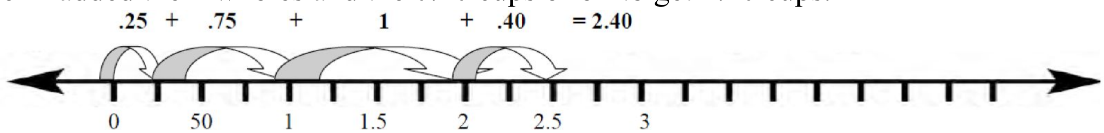


The solution is $3^{7}/_{10}$ or 3.7.
Example:
A recipe for a cake requires 1.25 cups of milk, 0.40 cups of oil, and 0.75 cups of water. How much liquid is in the mixing bowl?

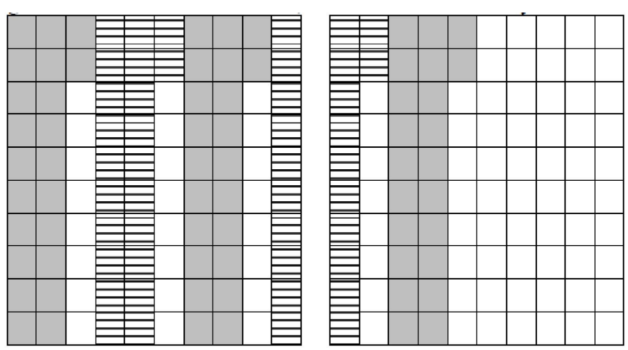
Student 1: 1.25 + 0.40 + 0.75
First, I broke the numbers apart. I broke 1.25 into 1.00 + 0.20 + 0.05. I left 0.40 like it was. I broke 0.75 into 0.70 + 0.05.
I combined my two 0.05's to get 0.10. I combined 0.40 and 0.20 to get 0.60. I added the 1 whole from 1.25. I ended up with 1 whole, 6 tenths, 7 more tenths, and another 1 tenth, so the total is 2.4.



Student 2: 1.25 + 0.40 + 0.75
I saw that the 0.25 in the 1.25 cups of milk and the 0.75 cups of water would combine to equal 1 whole cup. That plus the 1 whole in the 1.25 cups of milk gives me 2 whole cups. Then I added the 2 wholes and the 0.40 cups of oil to get 2.40 cups.

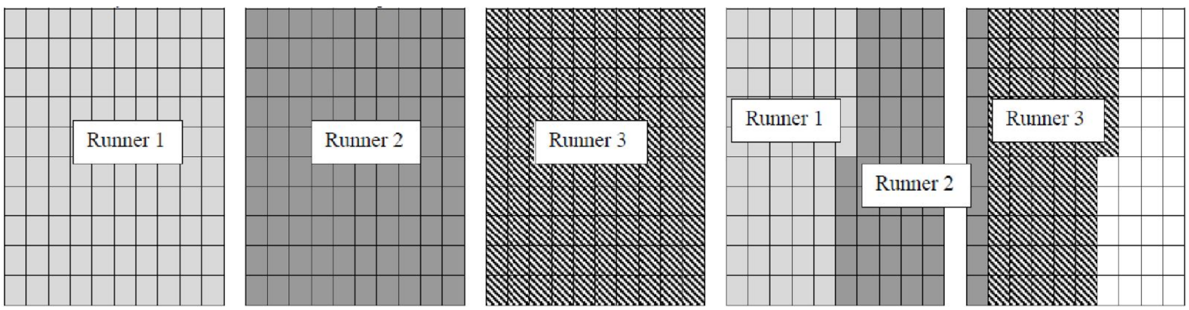


Example:
A gumball costs \$0.22. How much do 5 gumballs cost? Estimate the total, and then calculate. Was your estimate close?



I estimate that the total cost will be a little more than a dollar. I know that five 20's equal 100 and we have five 22's. I have 10 whole columns shaded and 10 individual boxes shaded to represent the five sets of 22 cents. The 10 columns equal 1 whole dollar. The 10 individual boxes equal 10 hundredths or 1 tenth. My answer is \$1.10.
My estimate was a little more than a dollar, and my answer was \$1.10. I was really close.

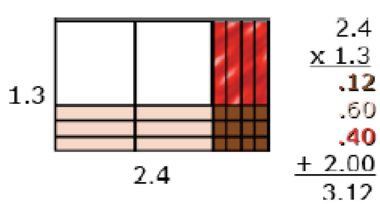
Example:
A relay race lasts 4.65 miles. The relay team has 3 runners. If each runner goes the same distance, how far does each team member run? Make an estimate, find your actual answer, and then compare them.



My estimate is that each runner runs between 1 and 2 miles. If each runner went 2 miles, that would be a total of 6 miles which is too high. If each runner ran 1 mile, that would be 3 miles, which is too low.
I used the 5 grids above to represent the 4.65 miles. I am going to use all of the first 4 grids and 65 of the squares in the 5th grid to represent the whole race. I have to divide the 4 whole grids and the 65 squares into 3 equal groups. I labeled each of the first 3 grids for each runner, so I know that each team member ran at least 1 mile. I then have 1 whole grid and 65 squares to divide up. Each column represents one-tenth. If I give 5 columns to each runner, that means that each runner has run 1 whole mile and 5 tenths of a mile. Now, I have 15 squares left to divide up. Each runner gets 5 of those squares. So each runner ran 1 mile, 5 tenths and 5 hundredths of a mile. I can write that as 1.55 miles.
My answer is 1.55 and my estimate was between 1 and 2 miles. I was pretty close.

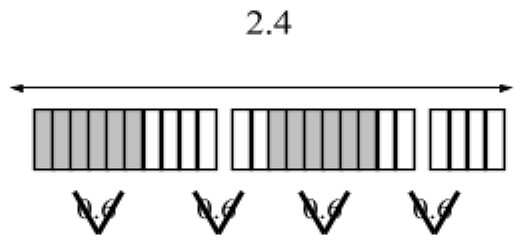
Example:
Models can be useful for illustrating products and quotients

Multiplication



Students should be able to describe the partial products displayed by the area model.
For example, "$^{3}/_{10}$ times $^{4}/_{10}$ is $^{12}/_{100}$.
$^{3}/_{10}$ times 2 is $^{6}/_{10}$ or $^{60}/_{100}$.
1 group of $^{4}/_{10}$ is $^{4}/_{10}$ or $^{40}/_{100}$.
1 group of 2 is 2."

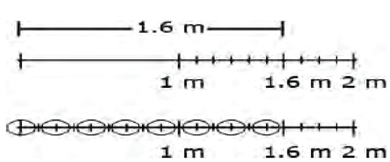
Division:
Finding the number in each group or share
Sherry has a board that is 2.4 yards long. She needs to share the wood equally with four people. How much wood will each person receive?
Students should be encouraged to apply a fair sharing model separating decimal values into equal parts.



$2.4 \div 4 = 0.6$

Division:
Finding the number of groups
Joe has 1.6 meters of rope. He has to cut pieces of rope that are 0.2 meters long. How many pieces can he cut?

Students could draw a segment to represent 1.6 meters. After marking off one whole meter, s/he would count in tenths to identify the 6 tenths more. Then, s/he would be able identify how many "2 tenths" are in the 6 tenths (3). The student can then extend the idea of counting by tenths to divide the one meter into tenths and determine that there are 5 more groups of 2 tenths. In all there are 8 of the 2 tenths in 1.6 meters.



Students might count groups of 2 tenths without the use of models or diagrams. Knowing that 1 can be thought of as $^{10}/_{10}$, a student might think of 1.6 as 16 tenths. Counting by 2s (2 tenths, 4 tenths, 6 tenths, …, 16 tenths), a student can count 8 groups of 2 tenths.
They can use their understanding of multiplication and think, "8 groups of 2 is 16, so 8 groups of $^{2}/_{10}$ is $^{16}/_{10}$ or $1^{6}/_{10}$."

<u>NF.5 Interpret multiplication as scaling (resizing), by:</u>

a. <u>Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication</u>

This standard calls for students to examine the magnitude of products in terms of the relationship between two situations.

Example 1:
Mrs. Jones teaches in a room that is 60 feet wide and 40 feet long. Mr. Thomas teaches in a room that is half

Example 2:
How does the product of 225 ´ 60 compare to the product of 225 ´ 30? How do you know? Since 30 is half

as wide, but has the same length. How do the dimensions and area of Mr. Thomas' classroom compare to Mrs. Jones' room? Draw a picture to prove your answer.	of 60, the product of 225 ´ 60 will be double or twice as large as the product of 225 ´ 30.

b. <u>Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence *a*/*b* = (*n* × *a*)/(*n* × *b*) to the effect of multiplying *a*/*b* by 1</u>

This standard asks students to examine how numbers change when we multiply by fractions. Students should have ample opportunities to examine both cases in the standard:

a) when multiplying by a fraction greater than one, the product increases
b) when multiplying by a fraction less the one, the product decreases.

Example:

3 x 2 ½ will be greater than 3, since we are multiplying by a fraction greater than 1

2 ½	2 ½	2 ½
7 ½		

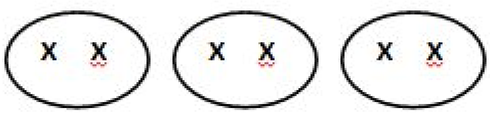
3 x ¼ will be less than 3, since we are multiplying by a fraction less than 1.

¼	¼	¼
¾		

NF.6 Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

This standard builds on all of the work done in this cluster. Students should be given ample opportunities to use various strategies to solve word problems involving the multiplication of a fraction by a mixed number. This standard could include fraction by a fraction, fraction by a mixed number, or mixed number by a mixed number.
Example:
Evan bought 6 roses for his mother. $^{2}/_{3}$ of them were red. How many red roses were there?

Student 1:
I divided the 6 roses into 3 equal groups. Then I counted how many roses were in 2 of the 3 groups. There are 4 red roses.

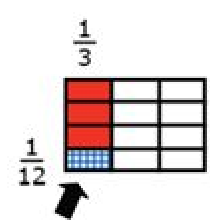


Student 2:
I used an equation to solve the problem. If there were 6 roses and 2/3 of them were red, I need to multiply 6 x 2/3 = 12/3= 4. There were 4 red roses.

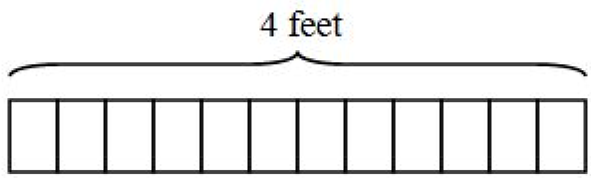
NF.7 Apply and extend previous understandings of division to divide unit fractions, by whole numbers and whole numbers by unit fractions

When students begin to work on this standard, it is the first time they are dividing with fractions. In 4th grade students divided whole numbers and multiplied a whole number by a fraction. The concept *unit fraction* is a fraction that has a one as a numerator. For example, the fraction $^{3}/_{5}$ is 3 copies of the unit fraction
$^{1}/_{5}$ ($^{1}/_{5} + {}^{1}/_{5} + {}^{1}/_{5} = {}^{3}/_{5}$)
Examples:

Four students sitting at a table were given $^{1}/_{3}$ of a pan of brownies to share. How much of the whole pan of brownies will each student get if they share it equally?
The diagram shows the $^{1}/_{3}$ pan of brownies divided into 4 equal shares with each share equaling $^{1}/_{12}$ of the pan. **1/3 ÷ 4 = 1/12**



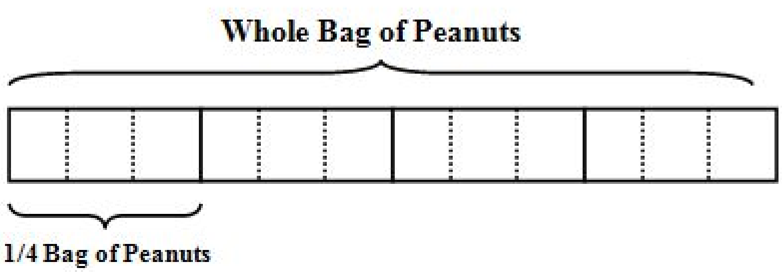
A piece of wood is 4 feet long. It needs to be cut into pieces that are 1/3 foot long. How many pieces of wood can be cut?



The diagram shows the 4 feet of wood divided into 1/3 foot equal parts. The board can be cut into 12 equal pieces. **4 ÷ 1/3 = 12**

a. <u>Interpret division of a unit fraction by a non-zero whole number, and compute such quotients.</u> ***<u>For example, create a story context for (1/3) ÷ 4, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that (1/3) ÷ 4 = 1/12 because (1/12) × 4 = 1/3.</u>***

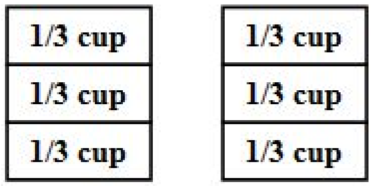
This standard asks students to work with story contexts where a unit fraction is divided by a non-zero whole number. Students should use various fraction models and reasoning about fractions.
Example:
You have $^{1}/_{4}$ of a bag of peanuts and you need to share that part of the bag among 3 people. How much of the whole bag does each person get?



When ¼ bag of peanuts is shared with 3 people, each person will receive 1/12 of the whole bag. ¼ ÷ 3 = 1/12

b. <u>Interpret division of a whole number by a unit fraction, and compute such quotients.</u> ***<u>For example, create a story context for 4 ÷ (1/5), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 × (1/5) = 4.</u>***

This standard calls for students to create story contexts and visual fraction models for division situations where a whole number is being divided by a unit fraction.
Example:
You have 2 cups of sugar. Each batch of cookies takes 1/3 cup of sugar. How many batches of cookies can you make?



There are six 1/3 cups in 2 cups of sugar. $2 \div 1/3 = 6$

c. <u>Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem.</u> ***<u>For example, how much chocolate will each person get if 3 people share ½ lb of chocolate equally? How many 1/3-cup servings are 2 cups of raisins?</u>***

Students will continue to model real world problems involving division of fractions by whole numbers and whole numbers by fractions (see above examples).